

# The Philosophy of Mathematics (and Infinity)

July 22 The Philosophy of Mathematics (and Infinity) - James Franklin

Philosophers have been fascinated by mathematics because it is the gold standard of knowledge - it's proved correct and stays that way. But what is it actually about - if biology is about life, what aspects of the world does mathematics study? Maybe it's just a language or method? The talk gives a realist view of mathematics as about a real subject matter, with examples from two areas: symmetry and infinity.

How many numbers are there? On your fingers, 1, 2, 3, 4, 5 ... on two hands ... 10, if it were summer and we had our thongs on ... 20 ... a million: that many dollars will just about get you distant water glimpses in Sydney real estate ... a billion: any serious hedge fund can lose that before breakfast ... a trillion: we're getting to the US arms budget or the number of neurons between all of us in the room ... after that, we may get to the number of atoms in the universe.

But the question wasn't: How many things are there? but, How many numbers are there? You might run out of things to count, but surely you aren't going to run out of numbers to count them with? The numbers themselves, and the possibility of always adding one to a number you've got and getting another one, seem to go on to infinity. And we seem to be quite certain of that.

We must emphasise that infinity is a very very long way away. By a kind of cognitive illusion like Everest appearing to be just over the near hills, we can easily think of infinity as just past a billion or a million. But it isn't. Here is a story to illustrate, coming from the Catholic childhood of legend where it was used to create fear of ending up for eternity in a place where you'd rather not be:

Imagine a huge steel sphere. Once in a thousand years, God sends down an eagle, which brushes the sphere with the tip of its wing. When the sphere is worn away to nothing, eternity is just beginning...

(Someone in the Philoagora audience quoted Woody Allen, "Eternity is very long, especially towards the end.")

Now, what about our knowledge that the numbers don't run out? Philosophically, it's very remarkable knowledge, because it's so far beyond sense experience yet so solid. For small numbers, you can see pairs, triples, 10-sided figures, arrays of 5 by 7 dots, etc, so there is some sort of perceptual access to small numbers. But infinity, how could you possibly know anything about that? Concentrating on that question leads us immediately into the heart of the philosophy of mathematics.

Well, how is such amazing knowledge possible? There are four ways you can go:

•1.

You can maintain that it's not possible; that we don't know that sort of thing. To be fair, you should maintain that it's an illusion to think we know about numbers beyond what we can count to. (Well, beyond what you have counted to.) That is heroically stubborn. If you tell me you believe that, I don't believe you.

•2.

You can maintain that we know those things about numbers because we made it that way. You could say that mathematics is just a human creation, and the way we use mathematical language, we just define numbers so that you can always add one. That can't be right either, since it would imply we could just as well make numbers some other way, e.g. our tribe could decide that it would be more convenient if  $2 + 2$  were 5, or minus 3.7. But the objectivity of mathematics forces itself on us: we can't make numbers and their relations with one another any other than the way they are. That is what makes mathematics such a good condom against the HIV of postmodernism.

•3.

You can take the Platonist option: which holds that there is an abstract realm beyond space and time, into which the soul has direct insight, possibly divinely implanted. That is certainly how it can feel with infinity or when discovering and understanding any mathematical truth, e.g. that the numbers not only go on to infinity but alternate odd and even,

that the primes among them thin out erratically but overall with a strictly logarithmic attenuation of density ... But the Platonist view does have the problem that it is very hard to fit with other things we believe philosophically, such as that our knowledge is implemented in a small finite chunk of wetware; it also leaves applied mathematics completely mysterious.

•4. We're running out of options ... but there is one more, the Aristotelian realist theory. I agree with it but don't maintain it's without problems. It says that we perceive the mathematical structure in complex parts of reality - their patterns, symmetry, order, continuity and the like - but we also have a faculty of understanding that can reflect on what we perceive and gain insight into the necessary connections between properties.

An example:

Imagine 3 balls in a row. Now align under them 3 more balls in a row. That's 2 lots of 3. But if you divide them vertically instead, there are 3 columns of 2. That's the same lot of things, so 3 2's equals 2 3's. Like so:

You now not only see that  $2 \times 3 = 3 \times 2$ , you understand why it must be so, and that it can't possibly be otherwise. Furthermore, you understand that it doesn't matter that we took small numbers like 2 and 3. We could take any numbers across and down, and conclude that  $m \times n = n \times m$  for any numbers  $m$  and  $n$ . Infinitely many truths, directly applicable in the real world, and known with certainty. That's mathematics.

Another kind of example of the mathematical structure of the real world that Aristotelian realism takes to be the basic subject matter of mathematics is symmetry. The simplest kind of symmetry is the bilateral symmetry that animal bodies have approximately and palindromes have exactly. But mathematicians are especially interested in objects with many symmetries. An example is the cube. Let's do a little mathematics to get a feel for it. We could use a real cube, but it's better to give our mental visualization facility (formerly "imagination") a workout - especially since we're all philosophers and prefer just sitting and thinking. Here's a set of cube visualization exercises that require you to use the multiple symmetry of the cube.

•1. Imagine looking down on an (opaque) cube from above one corner (vertex). How many corners can you see? (Including the ones outlined against the background?)

•2. Paint a wooden cube. Cut it into 3 each way (3 horizontally, 3 vertically, 3 vertically the other way). Of the 27 small cubes you've made, how many are painted on exactly 2 sides?

•3. How many rotations of a cube are there? (That is, rotations of a cube about an axis through its centre that leave the cube occupying the same space as it did to start with). This is harder, don't try it while driving ...

(Answers: 7; 12; 23 - or 24 if you count leaving it as is a rotation through zero degrees).

The philosophical point is that symmetry is a property at once very abstract and realizable in real things (physical things, but abstract things such as arguments can also have symmetry). Truths about it, such as those we just found about the cube, are objective and independent of humans, and are also accessible to our direct intellectual knowledge. That is the sort of thing mathematics, the science of structure or pattern, is about.

Further reading:

The "Sydney School" in the philosophy of mathematics

Interview with James Franklin on the philosophy of mathematics

Article 'Mathematical necessity and reality'.